U.G. 3rd Semester Examination - 2022

MATHEMATICS

[PROGRAMME]

Course Code : MATH-G-CC-T-03

(Real Analysis)

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

   a) Write the order properties of the system R of real numbers.

   b) Prove that the set of all integral multiples of 5 is enumerable.

   c) Define the least upper bound of a bounded set and obtain it for the set $A = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots\right\}$.

   d) Show that a non-empty finite set cannot be a neighbourhood of any point.

   e) Prove that the set $A = \{-1, 1, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \dots\}$ is neither open nor closed.

[Turn over]

f) Show that the sequence $\left\{\frac{3n-1}{n+2}\right\}$ is monotonic increasing and bounded.

g) Prove that the sequence $\left\{\frac{1}{n^p}\right\}$, where $p > 0$, is a null sequence.

h) Test for the convergence of the sequence $\{x_n\}$, where $x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!}, n \in N$.

i) State Leibnitz's Test on alternating series.

j) Define "conditional convergence" with an example.

k) Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n} x^n$.

l) Using Cauchy's General Principle for convergence show that harmonic series $\sum u_n = \sum \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} + \ldots \infty$ does not converge.

m) Prove that if a series $\sum u_n$ converges, then $\lim_{n\to\infty} u_n = 0$.

n) If $\sum a_n$ be a positive convergent series, then prove that $\sum {a_n}^2$ also converges.

o) Construct an example of a set no element of which lies between its upper and lower limits.

2. Answer any **four** questions: 5×4=20

a) **State and prove Cauchy's First Theorem on Limit.**

b) State and prove density property of the set of real numbers R.

c) Prove that the interior of a set $S \subseteq R$ is an open set.

d) Prove that if a sequence $\{x_n\}$ of real numbers is monotonic increasing and bounded above, then it converges to its exact upper bound.

e) Find the interval of convergence of the power series $x + \frac{x^2}{2\cdot 10} + \frac{x^3}{3\cdot 10^2} + \ldots + \frac{x^n}{n\cdot 10^{n-1}} + \ldots\infty$.

f) State and prove Comparison test for an infinite series of positive terms.

g) Prove that if S is a closed subset of R then its compliment S^c is open in R.

3. Answer any **two** questions: 10×2=20

a) i) Prove that the union of an enumerable number of enumerable sets is enumerable.

ii) Define derived set. With suitable examples show that arbitrary intersection of open sets may or may not be an open set.

6+(2+2)

b) i) State D'Alembert's Ratio Test for convergence or divergence of an infinite series of positive terms. Prove that the series $\sum_{n=1}^{\infty} \frac{n!2^n}{n^n}$ is convergent.

ii) State Raabe's test for convergence and divergence of an infinite series of positive terms. Examine the convergence of the series $1 + \frac{3}{7} + \frac{3 \cdot 6}{7 \cdot 10} + \frac{3 \cdot 6 \cdot 9}{7 \cdot 10 \cdot 13} + \ldots \infty$.

(2+3)+(2+3)

c) i) Show that the power series of $\tan^{-1}x$ is given by

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \infty \; (-1 \leq x \leq 1)$.

Hence derive that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \infty$.

ii) Show that

$$\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}} \right) = 1.$$

(5+2)+3

d) i) Prove that the sequence $\{x_n\}$ defined by $x_1 = \sqrt{6}$ and $x_{n+1} = \sqrt{6 + x_n}$ for $n \geq 1$, converges to 3.

ii) Show that $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$ by using Cauchy's second theorem on limit and hence show that $\lim_{n\to\infty} \frac{1}{n}\left(1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + \ldots + n^{\frac{1}{n}}\right) = 1$.

5+(3+2)

Government General Degree College, Chapra

1st Internal Assessment 2024

MATH-MI-T-01(Minor)

Semester I

Algebra & Analytical Geometry

Time : 30 minutes **Full Marks :10**

1. Answer **any two** questions from the following **(2 x 5= 10)**

a) If α, β be the roots of the equation $t^2 + 2t + 4 = 0$ and m is a positive integer , prove that $\alpha^m + \beta^m = 2^{m+1}\cos(\frac{2m\pi}{3})$

b) Two complex numbers z_1, z_2 are such that $|z_1 + z_2| = |z_1 - z_2|$ prove that amp z_1 and amp z_2 differ by $\frac{\pi}{2}$ or $\frac{3\pi}{2}$.

c) Solve the equation $2x^4 - 3x^3 - 3x^2 - 3x - 1 = 0$, one root being $1 + \sqrt{2}$.

d) Solve the equation $x^3 + 6x^2 + 11x + 6 = 0$, given that the roots are in arithmetic progression.

Government General Degree College, Chapra

Internal Examination

Semester-IV

B.Sc Programme

Sub: Mathematics (GCC) F.M: 15

Answer all the questions as per instruction

1. Express (5,2,1) as a linear combination of (1,1,0) and (3,0,1). 2

2. Find the basic feasible solution of the following system of equations

$$x_1 + 2x_3 = 1$$
$$x_2 + x_3 = 4, \quad \text{if} \quad x_1, x_2, x_3 \geq 0$$ 4

3. What is convex set? Discuss whether the following set is convex or not
X= $\{(x_1, x_2)/x_1 + 7x_2 = 5\}$ 3

4. Solve the following transportation problem 6

DESTINATION

ORIGIN						
	2	11	10	3	7	4
	1	4	7	2	1	8
	3	9	4	8	12	9
	3	3	4	5	6	

2ND Semester Internal Examination
Math - Sec - T - 02
Fuzzy Set Theory

F.M. - 10 Time : 30 min.

Answer any two questions:

1. Define fuzzy set and Convex fuzzy set.
   Show that a fuzzy set A on $\mathbb{R}$ is convex if and only if
   $$A(\lambda x_1+(1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$$
   for all $x_1, x_2 \in \mathbb{R}$ and for all $\lambda \in [0, 1]$. [2+3]

2. Define for a fuzzy set A, ${}^{\alpha}A$ and ${}^{\alpha+}A$.
   Let
   $$A(x) = \begin{cases} 1 & \text{when } x \leq 20 \\ \frac{35-x}{15} & \text{when } 20 < x < 35 \\ 0 & \text{when } x \geq 35 \end{cases}$$ [2+3]
   Find ${}^{\alpha}A$ and ${}^{\alpha+}A$ for any $\alpha \in \mathbb{R}$.

3. Let $A, B \in \mathcal{F}(X)$ and $\alpha \in [0,1]$.
   Prove that
   a. $A \subseteq B$ if and only if ${}^{\alpha}A \subseteq {}^{\alpha}B$
      $A \subseteq B$ if and only if ${}^{\alpha+}A \subseteq {}^{\alpha+}B$
   b. $A = B$ if and only if ${}^{\alpha}A = {}^{\alpha}B$ ~~and ${}^{\alpha+}A = {}^{\alpha+}B$~~
      if and only if ${}^{\alpha+}A = {}^{\alpha+}B$ [2.5 + 2.5]

Government General Degree College, Chapra

2^{nd} Semester Internal Examination 2024
MATHEMATICS (Major)
Course Code: MATH-T-02
Course title: Algebra-I

Full Marks: 15

Time: 40 minute

Answer any **THREE** of the following [$5 \times 3 = 15$]

1. State De Moivre's theorem and using it show that If n is an integer then $\left(\frac{1+\sin(\theta)+i\cos(\theta)}{1+\sin(\theta)-i\cos(\theta)}\right)^n = \cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$. [1+4=5 Marks]

2. Show that the ratio of the principal values of $(1+i)^{(1+i)}$ and $(1-i)^{(1-i)}$ is $\sin(\ln 2)+i\cos(\ln 2)$.

   [5 Marks]

3. Solve the equation $x^4 - x^3 + 2x^2 - 2x + 4 = 0$ one of whose root is $1+i$. [5 Marks]

4. Solve the equation $x^3 - 6x + 4 = 0$ by Cardan Method. [5 Marks]

5. Examine the solvability of following system of equation and hence solve it [5 Marks]

$$\begin{aligned} x+y+z &= 1 \\ 2x+y+2z &= 1 \\ x+2y+3z &= 0 \end{aligned}$$

Government General Degree College, Chapra

Internal Examination -2024

Semester-II (NEP)

Sub: Mathematics (MINOR) F.M: 15

Time : 40 Min.

Answer the given questions as per instruction (Any Five) $3 \times 5 = 15$

1. Find the modulus and amplitude of the complex number $z = 1 + i\tan\frac{3\pi}{5}$ 3
2. Show that the points representing the complex numbers (2 + 3 i), (4+ 6 i), (8 +12 i) in an argand plane are collinear. 3
3. Solve the given differential equation $x^3 - 9x^2 + 23x - 15 = 0$, whose roots are Arithmetic Progress. 3
4. Solve the equation $x^3 - 6x - 9 = 0$ by Cardan's method. 3
5. Is the following system of equations solvable? $x + y + z = 4, 2x + 5y - 2z = 3, x + 7y - 7z = 5$ 3
6. If, in a group G, $x^2 = e(identity\ element)$ for every $x \in G$, then prove that G is abelian. 3
7. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects angels between the other pair, prove that $pq = -1$ 3

Government General Degree College, Chapra

Internal Examination 2024

Semester-V

B.Sc Programme

Time: 40 min **Sub: Mathematics (DSE)** **F.M: 15**

Answer the given question as per instruction (any five) **3 x 5 =15**

1. Prove that the set D of all odd integers forms a commutative group with respect to the composition '*'defined by $a * b = a + b - 1$ for all $a, b \in D$. 3

2. Let (G,°) be a group. Define a mapping $f: G \rightarrow G$ by $f(x) = x^{-1}$, $x \in G$. prove that f is a bijection. 3
3. Define cyclic group. Find all the cyclic subgroup of the group (S, .) where S= $\{1, i, -1, -i\}$. 3
4. Prove that every group of prime order is cyclic. 3

5. Define the basis of vector space. Show that the set $S = \{(1,2,2), (2,1,2), (2,2,1)\}$ forms a basis of R^3. 3
6. Find the dimension of the subspace S of R^4 defined by $S = \{(x, y, z, w) \in R^4: x + y + z + w = 0\}$ 3
7. Find the eigen values of the following matrix

$$S = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$ 3

Government General Degree College, Chapra

3^{rd} Semester Internal Examination 2025
B.Sc Mathematics (Minor)
Course Code: MATH-MI-T-02
Course title: Calculus & Differential Equations

Full Marks: 15 Time: 30 minute

Answer any **FIVE** of the following [3 × 5 = 15]

1. Show that by $\epsilon - \delta$ definition: $\lim_{x \to 3} x^3 = 27$. [3 Marks]

2. If $y = \dfrac{1}{(ax+b)^n}$, find y_n. [3 Marks]

3. If $y = tan^{-1}x$, show that

   (i) $(1+x^2)y_1 = 1$, [1 Marks]

   (ii) $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$. [2 Marks]

4. Find the value of $\lim_{x \to 0} \left(\dfrac{sin(2x) + 7x^2 - 2x}{x^2(x+1)^2} \right)$. [3 Marks]

5. Find the reduction formula for $\int \sin^n x dx$ and using it find the value of $\int_0^{\frac{\pi}{2}} \sin^n x dx$. [3 Marks]

6. Solve the ode: $y' - \dfrac{2y}{x} = -x^2y^2$. [3 Marks]

Government General Degree College, Chapra

3^{rd} Semester Internal Examination 2025
B.Sc Mathematics (Major)
Course Code: MATH-SEC-T-03
Course title: Programming in C

Full Marks: 10

Time: 30 minute

Answer any **TWO** of the following $[5 \times 2 = 10]$

1. Write a short note on the following statements:
   while, do-while, for [5 Marks]

2. Write a flow chart to verify a given number is prime or not. [5 Marks]

3. Write a C program to verify a given year is leap year or not. [5 Marks]

4. What is the output of the following C program:

```
int main()
{
int a=300,b=500,c;
if(a > 300)
b = 200;
if(b > 300)
c = 100;
else
c = 50;
printf("\n%d\t%d\t%d", a, b, c) ;
return 0;
}
```

[5 Marks]

Government General Degree College, Chapra

Internal Examination, 2025

Semester-III

Full Marks: 15 **Sub: Mathematics (Major)** **Time: 40 min**

Answer the following questions as per instruction (**Any Five**): $5 \times 3 = 15$

1. Define supremum of a set. Determine the supremum and infimum of the following set

$$S = \{\frac{1}{m} + \frac{1}{n} / \, m, n \in N\}$$

2. Prove that the set of limit points of every sequence $\{x_n\}$ is a closed set.
3. If $0 \le x \le 1$, prove that the series $\sum \frac{x^n}{n^n}$ is convergent.
4. Prove that the sequence $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}$, ...converges 2.
5. Examine if $\lim_{x \to 0}(\sin\frac{1}{x} + x \sin\frac{1}{x})$ exist or not.
6. State the *intermediate value theorem* for continuous function and using it prove that there exist $x \in (0, \frac{\pi}{2})$ such that $x = \cos x$.
7. State Lagrange mean Value theorem and using it prove that $\sin x < x < \tan x \ in \ (0, \frac{\pi}{2})$